

# PERFORMANCE EVALUATION OF AN OBS NETWORK AS A TANDEM NETWORK OF IPP/M/W/W NODES

LINA BATTESILLI\*

*IBM Research  
Research Triangle Park, NC 27709, USA  
email: lina@battestilli.net*

HARRY PERROS

*Computer Science Department, North Carolina State University  
Raleigh, NC 27695-7534, USA  
email: hp@csc.ncsu.edu*

We propose an analytical method of calculating the burst loss probabilities in a tandem Optical Burst Switched (OBS) network with a *bursty* arrival process. We model the bursty arrival process as an Interrupted Poisson Process (IPP) and thus we are solving a tandem network of IPP/M/W/W loss nodes. We show how any traffic stream can be approximated as an IPP. Our performance evaluation of an OBS network shows that our analytical method approximates simulation results better than a Poisson arrival process.

## 1. Introduction

Most of the analytical models of Optical Burst Switched (OBS) Networks focus on a single OBS node. These models provide a limited insight about the overall performance of an OBS network. An analytical model of an OBS network is proposed in [1], where the OBS network is modeled by a network of loss nodes, each representing a link of  $W$  wavelengths. Bursts are assumed to arrive in a Poisson fashion and each burst occupies a single wavelength on each link along its source-destination path until it is lost or until it departs from the network.

Typically, the Poisson process is used to model the arrival traffic to a network because it is mathematically tractable. However, the Poisson process is *smooth* and the burst loss calculated is lower than for a more realistic *bursty* traffic. The term *bursty* traffic is not OBS specific and it

---

\*Corresponding Author, Tel.: +001-919-949-7682

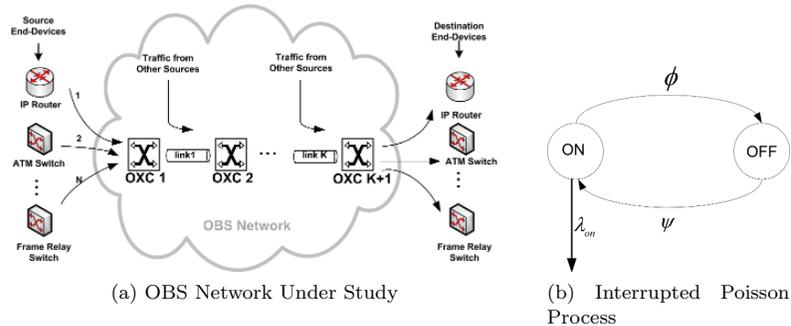


Figure 1.

does *not* refer to the fact that the arrival process is made of data bursts. In any network, the traffic is considered *bursty* if a large number of arrivals are followed by long idle periods.

The focus of this paper is the end-to-end performance evaluation of an OBS network with *bursty* traffic, where the bursts *dynamically* acquire and release wavelengths from link to link as they travel from their source to their destination. We use methods from teletraffic theory to obtain analytical results.

This paper is organized as follows. In Section 2 we describe the network under study and the proposed queueing network model. In Section 3 we set the bursty arrival process to be an Interrupted Poisson Process (IPP). In Section 4, we characterize the departure process from a loss node with IPP arrivals and exponential holding times. In Section 5 we show how any traffic stream can be approximated with an IPP. In Section 6 we propose an algorithm for analytically obtaining the blocking probabilities in an OBS transmission path with IPP arrivals. We conclude in Section 7.

## 2. Network Under Study

We study an OBS network, where the nodes are built from an Optical Cross Connect (OXC) and an electronic control unit. Two adjacent network nodes are linked by a single WDM link (fiber), which has  $W+1$  transmission wavelengths. The first  $W$  wavelengths are used for burst transmission while the  $(W+1)^{st}$  wavelength is used to transmit control information. Each OBS node has a full wavelength conversion capability, i.e., in the case of contention at an output port it can optically convert an optical signal from one wavelength to another. There are no fiber delay lines available at the network nodes and thus a burst is lost if it arrives at an output port where

all the wavelengths are busy.

In this OBS network we analyze the performance of a specific source-destination transmission path, made of  $K$  links. Therefore, we consider  $(K + 1)$  network nodes connected in tandem, as shown in Figure 1a. We consider the traffic flow from left to right. The end-devices, linked to OXC 1, transmit bursts to a number of end-devices, linked to OXC  $(K + 1)$ . We refer to the traffic generated from the transmitting end-devices as the *cross traffic*. In addition, to the cross traffic we consider traffic generated by other sources in the OBS network. This traffic arrives at the intermediate links of the considered path. We refer to this traffic as the *local* burst traffic. The local traffic is routed toward the same destination end-devices as the cross traffic.

This OBS network can be modeled as a tandem queueing network of IPP/M/W/W loss nodes, where each loss node represents one of the WDM links. We assume bursty arrival process, which we describe in Section 3.

### 3. Interrupted Poisson Process

For the bursty arrival process we choose a 2-state Markov Modulated Poisson Process (MMPP), which modulates between two exponentially distributed states, an ON and an OFF state. The transition rates for the ON and OFF states are respectively  $\phi$  and  $\psi$ , see Figure 1b. While in the ON state, the process generates Poisson arrivals with rate  $\lambda_{on}$  whereas in the OFF state there are *no* burst arrivals. This process is known as the Interrupted Poisson Process (IPP). Intuitively, an IPP is bursty because the arrivals are batched together during the ON period and no arrivals occur during the OFF period. The IPP becomes burstier when long OFF periods are followed by short ON periods with a large arrival rate  $\lambda_{on}$ . An IPP is uniquely characterized through the parameters  $\phi$ ,  $\psi$  and  $\lambda_{on}$ .

#### 3.1. Interarrival Time Description of an IPP

We first characterize the burstiness of an IPP traffic by using the interarrival time. We calculate the squared coefficient of variation  $c^2$ , which is the ratio of the variance to the squared mean of the interarrival time. The  $c^2$  is a dimensionless number that represents the relative variation of interarrival times about the average. A small  $c^2$  represents interarrival times that concentrate mostly around the mean. For a Poisson process the  $c^2$  is equal to 1. For an IPP the  $c^2$  is always greater than 1 if  $\phi > 0$  [2]. The mean, variance and  $c^2$  of an IPP are ( see [3]):

$$mean = \frac{\phi + \psi}{\lambda_{on}\psi}, \quad var = \frac{2\lambda_{on}\phi + (\phi + \psi)^2}{(\lambda_{on}\psi)^2}, \quad c^2 = 1 + \frac{2\lambda_{on}\phi}{(\phi + \psi)^2}.$$

In addition, we define the *coefficient of burstiness*  $r$  of an IPP to be the ratio of the average ON time to the sum of the average ON and OFF time:

$$r = \frac{1/\phi}{1/\phi + 1/\psi} = \frac{\psi}{\psi + \phi}, \quad 0 < r < 1. \quad (1)$$

An IPP is more bursty if  $r$  is close to 0 and it gets less bursty as  $r$  approaches 1.

Let us denote the IPP average arrival rate with  $\lambda_{avg} = 1/mean$ . Now given  $\lambda_{avg}$ ,  $c^2$  and  $r$  we define an unique IPP and determine its three parameters:

$$\lambda_{on} = \frac{\lambda_{avg}}{r}, \quad \phi = \frac{2\lambda_{avg}(1-r)^2}{r(c^2-1)}, \quad \psi = \frac{2\lambda_{avg}(1-r)}{c^2-1}, \quad c^2 > 1. \quad (2)$$

We have simulated IPP a number of times and observed that, as expected, the IPPs with higher  $c^2$  have higher blocking probability.

### 3.2. Infinite Server Description of an IPP

We now describe an IPP process using the Infinite Server Effect (ISE) principle from teletraffic theory. Using the ISE principle, a traffic stream is offered to an infinite-server system in order to calculate the mean number  $m$  and the variance  $v$  of the number of busy servers. Using the work of of A. Kuczura [4], we obtain that:

$$m = M_1 = \lambda_{on} \left( \frac{\psi}{\phi + \psi} \right), \quad v = \lambda_{on}^2 \left( \frac{\psi + 1}{\phi + \psi + 1} \right) \left( \frac{\psi}{\phi + \psi} \right) - m^2 + m. \quad (3)$$

## 4. Departure Process from an IPP/M/W/W Loss Node

Next, we characterize the departure process from an IPP/M/W/W loss node, shown in Figure 2. An IPP process, characterized by its  $m_{in}$  and  $v_{in}$ , is offered to loss node 1 which has  $W$  servers. Using the ISE principle, the departure process from loss node 1 is then offered to the infinite server node 2 in order to estimate its  $m_{out}$  and  $v_{out}$ .

Our method of characterizing the departure process from an IPP/M/W/W loss node follows the Rajaratnam and Takawira [5,6] approach. Let us begin by defining the joint probability distribution  $p_{a,w,c}$ ,

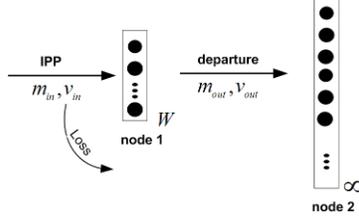


Figure 2.: Departure Process from an IPP/M/W/W loss node

where (1)  $a$  is the state of the IPP,  $a = 1$  is for the ON state and  $a = 0$  is for the OFF state (2)  $w$ ,  $0 \leq w \leq W$ , is the number of busy servers in node 1 (3)  $c$ ,  $0 \leq c < \infty$ , is the number of busy servers in node 2.

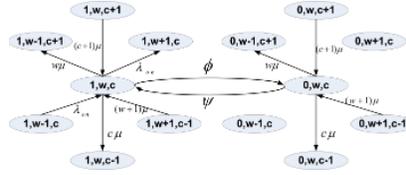


Figure 3.: Transition Rate Diagram for  $p_{a,w,c}$

The rate transitions for any state of this distribution are shown in Figure 3.

The partial binomial moments of this probability distribution given by the expression ( see [2] ):

$$\beta_{awj} = \sum_{c=j}^{\infty} \binom{c}{j} p_{a,w,c}, \quad a = 0, 1; \quad 0 \leq w \leq W \quad (4)$$

and the  $j^{th}$  binomial moment is  $\beta_j = \sum_{w=0}^W [\beta_{1wj} + \beta_{0wj}]$ .

Therefore, the first binomial moment is

$$\beta_1 = \sum_{w=0}^W \left[ \sum_{c=1}^{\infty} c (p_{1wc} + p_{0wc}) \right] = m_{out}.$$

The second binomial moment is:

$$\beta_2 = \sum_{w=0}^W [\beta_{1w2} + \beta_{0w2}] = \frac{1}{2} \sum_{w=0}^W \left[ \sum_{c=1}^{\infty} c^2 (p_{1wc} + p_{0wc}) - \sum_{c=1}^{\infty} c (p_{1wc} + p_{0wc}) \right]$$

and  $v_{out} = 2\beta_2 - \beta_1^2 + \beta_1$ .

So, in order to compute  $m_{out}$  and  $v_{out}$ , we only need to determine  $\beta_1$  and  $\beta_2$ . We have found that these two parameters are given by (see [7] for details):

$$\beta_1 = \sum_{w=0}^W w(\beta_{1w0} + \beta_{0w0}) \quad \text{and} \quad \beta_2 = \frac{1}{2} \sum_{w=0}^W w(\beta_{1w1} + \beta_{0w1})$$

For  $\beta_1$  we need  $\beta_{1w0}$  and  $\beta_{0w0}$ . We have that  $\beta_{1w0} = \sum_{c=0}^{\infty} \binom{c}{0} p_{1wc}$ , which is simply the steady-state probability that there are  $w$  busy wavelengths in node 1 and the IPP is in the ON state. Similarly,  $\beta_{0w0}$  is the steady state probability that there are  $w$  busy wavelengths in node 1 and the IPP is in the OFF state. So, in order to find  $\beta_1$  we need the steady-state probabilities of node 1. If we consider only node 1, the corresponding probability distribution is  $p_{a,w}$ ,  $a = 0, 1$  and  $0 \leq w \leq W$ . The transition rate diagram for node 1 is shown in Figure 4. We solve for the steady-state probabilities at node 1 numerically. There is a total of  $2(W+1)$  states and thus the numerical solution requires multiplication of the  $2(W+1) \times 2(W+1)$  rate matrix.

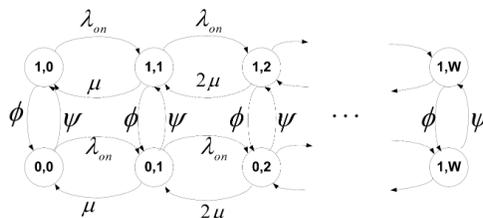


Figure 4.: Transition Rate Diagram for an IPP loss node

Next, we find  $\beta_2$  numerically by solving for  $\beta_{aw1}$ ,  $a = 0, 1$ ;  $0 \leq w \leq W$ , using the modified system of local balance equations. Now, that we have  $\beta_1$  and  $\beta_2$  we can calculate  $m_{out}$  and  $v_{out}$  of the departure process.

### 5. Modeling Any Traffic Stream with a Given $m$ and $v$ as an IPP

Any traffic stream with a given  $m$  and  $v$  can be modeled by an IPP. In [4], Kuczura presents a three-moment match and a two-moment match for modeling any traffic stream as an IPP. In this paper, we use the two-moment match. An IPP has three parameters but we only set the first two moments

and thus it is necessary to fix one of the parameters  $\phi$ ,  $\psi$  or  $\lambda_{on}$ . Kuczura uses the Equivalent Random Method (ERM) [8] from teletraffic theory, also known as the Wilkinson's method [9]. Using Rapp's approximation [10] we first set  $\lambda_{on}$  to be:

$$\lambda_{on} = v + 3 \frac{v}{m} \left( \frac{v}{m} - 1 \right). \quad (5)$$

then the other two IPP parameters can be obtained by:

$$\phi = \left( \frac{\lambda_{on}}{m} - 1 \right) \psi, \quad \psi = \frac{m}{\lambda_{on}} \left( \frac{\lambda_{on} - m}{\frac{v}{m} - 1} - 1 \right).$$

## 6. The Algorithm

Now, let us recall the problem at hand, i.e., IPP bursty arrivals offered to an OBS path with large number of  $W$  wavelengths per link. We begin with node 1. The arrival traffic to node 1 is an IPP with given  $c_1^2$ ,  $r_1$  and  $\lambda_{avg_1}$  and we obtain  $m_1$  and  $v_1$ . Next, we solve for the steady state probability  $\pi_1$  numerically from which we can also calculate the blocking probability at node 1. In order to estimate the arrival process to node 2, we then need to characterize the departure process from node 1, i.e., calculate  $m_1^{out}$  and  $v_1^{out}$ . This is done using the technique from Section 4. Assuming that the local arrivals at the intermediate links are also bursty, we model them also with an IPP characterized by  $c_{loc}^2$ ,  $r_{loc}$  and  $\lambda_{avg_{loc}}$ . Again,  $m_{loc}$  and  $v_{loc}$  are obtained using (3). Therefore, the arrival rate to node 2 is

$$m_2 = m_1^{out} + m_{loc}, \quad v_2 = v_1^{out} + v_{loc},$$

where the variances are added since the cross traffic and the local traffic are independent.

Next, we construct the IPP arrival process to node 2 that corresponds to a traffic stream characterized by the ISE parameters  $m_2$  and  $v_2$ . Now, we can solve numerically for the steady state probabilities at node 2, i.e,  $\pi_2$ . The blocking probability at node 2 can be obtained from  $\pi_2$ . The same steps are repeated for the rest of the nodes in the queueing network.

In [7] we provide an extensive numerical study to illustrate that, if the arrival process is bursty, our analytical algorithm produces results for the burst loss probabilities better than a simple Poisson approximation.

## 7. Conclusions

In this paper, we presented a novel analytical approach for the analysis of a tandem OBS network, assuming that bursts arrive according to an Interrupted Poisson Process (IPP). The queueing network was analyzed by a single-node decomposition, whereby each node was studied in isolation as a loss node with an IPP arrival, i.e., as an IPP/M/W/W node. Based on our numerical experiments, we found that if the arrival process is characterized as bursty, our analytical algorithm approximates the burst loss probabilities better than a simple Poisson approximation.

## References

- [1] Z. Rosberg, Hai Le Vu, M. Zukerman, and J. White. Performance analyses of optical burst-switching networks. *IEEE Journal on Selected Areas in Communications*, 21(7), Sept. 2003.
- [2] Andre Girard. *Routing and Dimensioning in Circuit-Switched Networks*. Addison-Wesley, 1990.
- [3] Wolfgang Fischer and Kathleen Meier-Hellstern. The markov-modulated poisson process (mmp) cookbook. *Perform. Eval.*, 18(2):149–171, 1993.
- [4] Anatol Kuczura. The interrupted poisson process as an overflow process. *The Bell System Technical Journal*, 52(3):437–448, March 1973.
- [5] Myuran Rajaratnam and Fambirai Takawira. Nonclassical traffic modeling and performance analysis of cellular mobile networks with and without channel reservation. *IEEE Transactions on Vehicular Technology*, 49(3):817–834, May 2000.
- [6] Myuran Rajaratnam and Fambirai Takawira. Handoff traffic characterization in cellular networks under nonclassical arrivals and service time distributions. *IEEE Transactions on Vehicular Technology*, 50(4):954–970, July 2001.
- [7] Tzvetelina (Lina) Battestilli. *Performance Analysis of Optical Burst Switched Networks with Dynamic Simultaneous Link Possession*. PhD thesis, NCSU, 2005.
- [8] Robert B. Cooper. *Introduction to queueing theory*. North Holland, 2 edition, 1981.
- [9] R. I. Wilkinson. Theories of toll traffic engineering in the usa. *The Bell System Technical Journal*, 35(2):421–514, 1956.
- [10] Y. Rapp. Planning of junction network in a multiexchange area. *Ericsson Technics*, (2):187–240, 1965.